C. U. SHAH UNIVERSITY
Winter Examination-2019

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT1   **Branch: Diploma (All)**

Semester : 2   **Date : 12/09/2019**   **Time : 02:30 To 05:30**   **Marks : 70**

Instructions:
 (1) Use of Programmable calculator & any other electronic instrument is prohibited.
 (2) Instructions written on main answer book are strictly to be obeyed.
 (3) Draw neat diagrams and figures (if necessary) at right places.
 (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**

a) If A(− 5, 7) and B(7, − 2) then AB = _____.
   (A) 15   (B) 169   (C) $\sqrt{29}$ (D) None of these

b) If A(1, 7) and B(3, 3) are the given points, then the midpoint of AB is _____.
   (A) $(-2,5)$ (B) $(5,-2)$ (C) $(2,5)$ (D) $(5,2)$

c) y – intercept of line 2x − 6y + 4 = 0 is _____.
   (A) −2/3   (B) 2/3   (C) −2   (D) 2

d) Radius of the circle $x^2 + y^2 = 25$ is _____ .
   (A) 5   (B) 25   (C) 5/2   (D) None of these

e) $\lim\limits_{x \to 1} \dfrac{x^3 - 1}{x - 1} =$ _____
   (A) − 1   (B) 1   (C) 0   (D) 3

f) $\lim\limits_{x \to 0} \dfrac{\tan x}{x} =$ _____
   (A) − 1   (B) 0   (C) 1   (D) None of these

g) $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x} =$ _____
   (A) $\log_a e$ (B) $\log_e a$ (C) e (D) 1

h) $\dfrac{d(\cos ecx)}{dx} =$ _____
   (A) $\tan x$ (B) $\sec^2 x$ (C) $-\tan x$ (D) $-\cos ecx \cot x$

i) $\dfrac{d(x)}{dx} =$ _____
   (A) $\dfrac{x^2}{2}$ (B) $\dfrac{1}{x}$ (C) 0   (D) 1

j) If $f(x) = \log \sqrt{x^2 + 1}$ then $f'(0) =$ _____.

(A) $\dfrac{1}{2}$ (B) 1   (C) 2   (D) 0

k) $\dfrac{d\left(e^{5x}\right)}{dx} =$ _____

(A) $5e^x$ (B) e^x (C) $\dfrac{e^{5x}}{5}$ (D) $5e^{5x}$

l) $\displaystyle\int \dfrac{1}{x}\, dx =$ _____

(A) $\log x + c$ (B) $e^x + c$ (C) 1   (D) 0

m) $\displaystyle\int \dfrac{1}{\sqrt{a^2 - x^2}}\, dx =$ _____

(A) $\cot^{-1} \dfrac{x}{a} + c$ (B) $\tan^{-1} \dfrac{x}{a} + c$ (C) $\cos^{-1} \dfrac{x}{a} + c$ (D) $\sin^{-1} \dfrac{x}{a} + c$

n) $\displaystyle\int_0^1 \dfrac{2}{1 + x^2}\, dx =$ _____

(A) π (B) $\dfrac{\pi}{4}$ (C) $\dfrac{\pi}{2}$ (D) None of these

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	a)	Find coordinates of the points of trisection of the line segment joining points A(4, 5) and B(13, − 4).	**(5)**
	b)	Find the equation of line perpendicular to line $4x - y + 5 = 0$ and passing through $(1, -2)$.	**(5)**
	c)	Prove that if $f(x) = \log\left(\dfrac{x-1}{x}\right)$ then prove that $f(x) + f(-x) = f(x^2)$.	**(4)**

Q-3		**Attempt all questions**	**(14)**
	a)	Prove that $\displaystyle\lim_{x \to 3} \dfrac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}} = \dfrac{\sqrt{35}}{5}$	**(5)**
	b)	Evaluate: $\displaystyle\lim_{x \to (-2)} \dfrac{x^3 + 2x^2 + x + 2}{x^2 + x - 2}$	**(5)**
	c)	Show that the points A(1, 2), B(2, 3) and C(0, 5) are the vertices of a right angled triangle.	**(4)**

Q-4		**Attempt all questions**	**(14)**
	a)	Find equation of a circle passing through points (1, 0), (0, 1) and (0, 0).	**(5)**
	b)	Find derivative of $y = 3^{4x}$ using definition.	**(5)**
	c)	Find $\dfrac{dy}{dx}$ if $y = \dfrac{\sin(\log x)}{x}$.	**(4)**

Q-5		**Attempt all questions**	**(14)**

a) Evaluate: $\lim\limits_{x\to 0}\dfrac{2\left(5^{x}\right)+3\left(2^{x}\right)-5}{x}$ **(5)**

b) The equation of motion of a particle is $S=2t^{3}+3t^{2}-12t+5$. **(5)**
(i) Find velocity at $t=0$. (ii) Find acceleration at $t=1$.

c) Find $\dfrac{dy}{dx}$ if $y=x^{\sin^{3}x}$. **(4)**

Q-6 **Attempt all questions** **(14)**

a) Evaluate: $\displaystyle\int\dfrac{x^{4}+x^{2}+1}{x^{2}+1}dx$ **(5)**

b) If $y=e^{x}\sin x$ then prove that $\dfrac{d^{2}y}{dx^{2}}-2\dfrac{dy}{dx}+2y=0$. **(5)**

c) Evaluate: $\displaystyle\int x\log x\,dx$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Find $\dfrac{dy}{dx}$ if $y=\log\left(\dfrac{\sin x}{1+\cos x}\right)$. **(5)**

b) Evaluate: $\displaystyle\int\dfrac{e^{x}\left(1+x\right)}{\cos^{2}\left(xe^{x}\right)}dx$ **(5)**

c) Evaluate: $\displaystyle\int\dfrac{\left(1-3x\right)^{2}}{x^{3}}dx$ **(4)**

Q-8 **Attempt all questions** **(14)**

a) Prove that $\displaystyle\int_{0}^{\frac{\pi}{2}}\dfrac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx=\dfrac{\pi}{4}$. **(5)**

b) Find the volume of sphere of radius r. **(5)**

c) Find the equation of circle having centre $(1,1)$ and passing through $(-2,4)$. **(4)**